Conical finite element applicable for curved panels

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Abstract

A semi-analytical model for the non-linear analysis of unstiffened laminated composite panels using the Ritz method and the Classical Laminated Plate Theory is proposed. A matrix notation is used to formulate the problem using the Donnell’s equations. The approximation functions proposed are capable to simulate a general load case with applied membrane distributed forces and bending moments. Solutions for linear static, linear buckling and non-linear buckling analyses are presented and verified using finite elements. The integration of the linear stiffness matrices is analytical while the non-linear terms are integrated numerically using a two-dimensional trapezoidal rule.

Keywords: Ritz method, linear buckling, linear static, non-linear analysis, composite, panels

# Kinematic equations

Kinematic equations (CLPT).

|  |  |  |
| --- | --- | --- |
|  |  | (1) |

Kinematic equations (FSDT).

|  |  |  |
| --- | --- | --- |
|  |  | (2) |

In order to apply the natural coordinate system of FIG::: FIXME

NATURAL COORDINATE SYSTEM

Fig. 1: Cone/cylinder coordinate system and geometric variables

Defining a

|  |  | (3) |
| --- | --- | --- |

Natural coordinates (CLPT):

|  |  | (4) |
| --- | --- | --- |

The variation of the strain vector can be written as (CLPT):

|  |  | (5) |
| --- | --- | --- |

Natural coordinates (FSDT):

|  |  | (6) |
| --- | --- | --- |

The variation of the strain vector can be written as (FSDT):

|  |  | (7) |
| --- | --- | --- |

# Finite elements

* 1. Conical element with 8 nodes

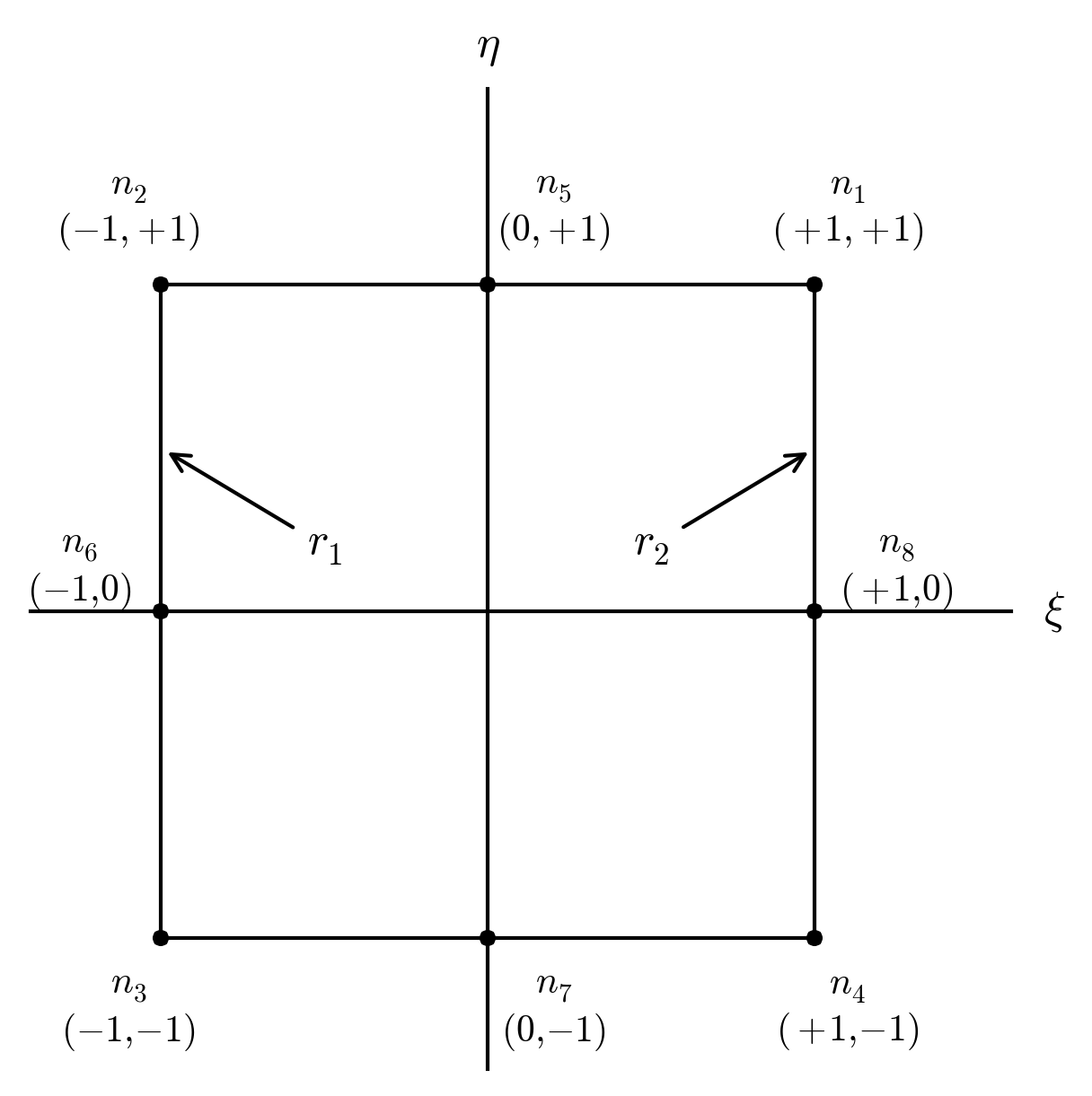


Fig. 2: Cone/cylinder coordinate system and geometric variables

|  |  | (8) |
| --- | --- | --- |

Quadrature rule using points, as suggested by Bathe (1996) [1]:

Table 1: Integration points (adapted from Ref. [1], Table 5.6)

|  |  |  |  |
| --- | --- | --- | --- |
| Point |  |  |  |
| 1 |  |  |  |
| 2 |  |  |  |
| 3 |  |  |  |
| 4 |  |  |  |
| 5 | 0 |  |  |
| 6 |  | 0 |  |
| 7 | 0 |  |  |
| 8 |  |  |  |
| 9 | 0 | 0 |  |

The stiffness matrices are integrated numerically where the integrands are evaluated at each integration point:

|  |  | (8) |
| --- | --- | --- |

with:

|  |  | (8) |
| --- | --- | --- |

with:

|  |  | (8) |
| --- | --- | --- |

The displacements are approximated as:

|  |  | (8) |
| --- | --- | --- |

The coordinates are approximated as:

|  |  | (8) |
| --- | --- | --- |

Table 2: Material properties

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| Material name | Reference | (GPa) | (GPa) |  | (GPa) | (GPa) | (GPa) |
| Geier2002 | [2] | 123.55 | 8.708 | 0.319 | 5.695 | 5.695 | 5.695 |
| DegCocomat | [3] | 142.50 | 8.700 | 0.28 | 5.100 | 5.100 | 5.100 |

# Conclusions

A semi-analytical model has been proposed for the non-linear analysis of simply supported laminated composite cylinders and cones under axial, torsion and pressure loads, using the Ritz method, the Classical Laminated Plate Theory and both Donnell’s and Sanders’ equations. The evaluated results for linear static, linear buckling and non-linear buckling are in close agreement with finite element predictions and the elephant’s foot effect has been successfully modeled making the proposed model applicable to identify this failure mode.

For linear buckling analysis it was shown how the orthotropic assumption can limit the prediction of linear buckling modes for composite structures, and the proposed model could appropriately take into account the shear-tension, torsion-tension and torsion-bending couplings.

Despite the results using Donnell’s and Sanders’ equations are in agreement with Simitses et al. (1985) [4] and Goldfeld [5] [6], where the Sanders’ model gives a buckling load which is smaller or equal the Donnell’s model, in the present study the use of Sanders’s equation is not justifiable, since it significantly increases the computational cost.

The proposed implementation leads to faster linear analyses when compared to finite elements but the non-linear analyses are considerably slower, and the bottleneck is currently the integration of the non-linear matrices , and . Therefore, further studies regarding the implementation should focus on this point. All the presented models and algorithms are publicly available in Ref. [7].

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| --- | --- |
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1. Geometric stiffness matrix for the Donnell’s and Sanders’ equations

Recalling from Eqs. **Fehler! Verweisquelle konnte nicht gefunden werden.** and **Fehler! Verweisquelle konnte nicht gefunden werden.** that , since does not depend on the Ritz constants, the first integral of Eq. **Fehler! Verweisquelle konnte nicht gefunden werden.** can be expanded using the Donnell’s equations as:

|  |  |  |
| --- | --- | --- |
|  |  | (A.1) |